

Thermalization of quark-gluon matter by 2-to-2 and 3-to-3 elastic scatterings

Xiao-Ming Xu

Department of Physics, Shanghai University, Baoshan, Shanghai 200444, China

E-mail: xmxu@xmxucao.sina.net

Abstract. Thermalization of quark-gluon matter is studied with a transport equation that includes contributions of 2-to-2 and 3-to-3 elastic scatterings. Thermalization time is related to the squared amplitudes for the elastic scatterings that are calculated in perturbative QCD.

PACS numbers: 24.85.+p;12.38.Mh;12.38.Bx;25.75.Nq

1. Introduction

For a nonequilibrium partonic system, two questions must be answered. One is that whether or not a thermal state can be established; another is that how fast thermalization of the system goes. The H-theorem ensures that the system evolves into a thermal state as a result of 2-body elastic scatterings [1]. In a system where 3-body elastic scatterings are important, a transport equation needs a term that reflects the 3-body elastic scatterings.

$$\begin{aligned}
 & \frac{\partial f_1}{\partial t} + \mathbf{v}_1 \cdot \nabla_{\mathbf{r}} f_1 \\
 &= -\frac{g}{2E_1 g_{22}} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \\
 & \quad \times |\mathcal{M}_{2 \rightarrow 2}|^2 [f_1 f_2 (1 \pm f_3)(1 \pm f_4) - f_3 f_4 (1 \pm f_1)(1 \pm f_2)] \\
 & \quad - \frac{g^2}{2E_1 g_{33}} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \frac{d^3 p_5}{(2\pi)^3 2E_5} \frac{d^3 p_6}{(2\pi)^3 2E_6} \\
 & \quad \times (2\pi)^4 \delta^4(p_1 + p_2 + p_3 - p_4 - p_5 - p_6) |\mathcal{M}_{3 \rightarrow 3}|^2 \\
 & \quad \times [f_1 f_2 f_3 (1 \pm f_4)(1 \pm f_5)(1 \pm f_6) - f_4 f_5 f_6 (1 \pm f_1)(1 \pm f_2)(1 \pm f_3)]
 \end{aligned} \tag{1}$$

where g is the color-spin degeneracy factor, $g_{22} = n'_{\text{out}}!$ and $g_{33} = n_{\text{in}}! n_{\text{out}}!$ where n'_{out} (n_{out}) is the number of identical final partons of 2-to-2 (3-to-3) scatterings and n_{in} for the 3-to-3 scatterings is the number of identical initial partons except the parton in the distribution function f_1 . $|\mathcal{M}_{2 \rightarrow 2}|^2$ and $|\mathcal{M}_{3 \rightarrow 3}|^2$ represent squared amplitudes of 2-to-2 and 3-to-3 parton scatterings, respectively.

Define

$$H(t) = \int d\vec{r} d\vec{p}_1 f(\vec{p}_1, \vec{r}, t) \ln f(\vec{p}_1, \vec{r}, t). \quad (2)$$

$H(t)$ is negative for $f < 1$ and $H(t)$ always decreases with increasing time until it is independent of time because f obeys the transport equation. When $H(t)$ does not rely on time, the distribution function takes a form of thermal distribution. Such reasoning proves that the system satisfying (1) is driven toward a thermal state by both 2-body and 3-body elastic scatterings. For a system that is governed by 2-body and 3-body elastic scatterings, the first question is answered.

The answer to the second question depends on the squared amplitudes $|\mathcal{M}_{2\rightarrow 2}|^2$ and $|\mathcal{M}_{3\rightarrow 3}|^2$ which relate to dynamical processes. The very simple, instructive but unrealistic case is that the squared amplitudes are constants. Even in the case the integrals on the right-hand side of the transport equation are generally not zero. An initial distribution function is anisotropic in momentum space for a nonequilibrium system. If the two squared amplitudes are zero, the distribution function remains unchanged. Then thermalization is never possible. If the two squared amplitudes take small values, the distribution function changes slowly with increasing time and thermalization is slow. Large values of the two squared amplitudes will cause large change of the distribution function and the corresponding thermalization goes fast. Even though the squared amplitudes are constants, we still need to numerically solve the transport equation to get thermalization time which is the difference of the time when the initial distribution is given and the time when the equation produces an isotropic momentum distribution. Therefore, how fast a system thermalizes depends on dynamical processes like elastic scatterings and anisotropy of initial distribution.

For a realistic case of interest, quark-gluon matter created in ultra-relativistic heavy-ion collisions, the squared amplitudes for 2-to-2 and 3-to-3 parton elastic scatterings have complicated dependences on parton momenta. Since the elastic scatterings take place among gluons, quarks and antiquarks with the same and/or different flavors, the study of how fast quark-gluon matter thermalizes becomes a very difficult task. However, from the discussion in the last paragraph, we immediately realize that thermalization of quark-gluon matter goes faster and faster if more and more types of parton elastic scatterings are included in $|\mathcal{M}_{2\rightarrow 2}|^2$ and $|\mathcal{M}_{3\rightarrow 3}|^2$. A relevant change of thermalization time is shown by our recent works [2, 3, 4] that have taken into account the four types of 3-to-3 elastic scatterings: quark-quark-quark, quark-quark-antiquark and quark-antiquark-antiquark for quark matter and gluon-gluon-gluon for gluon matter. The 3-to-3 elastic scatterings, thermalization time and summary are given in the next two sections.

2. 3-to-3 Elastic scatterings

The 3-to-3 quark scatterings at the lowest order are shown in figure 1 [2]. The left diagram indicates two-gluon-exchange induced scattering and the right involves triple-

gluon coupling. Initial (final) quarks may have the same or different flavors. While two or three initial (final) quarks have the same flavor, exchanges of quarks must be taken into account.

Two typical processes of 3-to-3 quark-quark-antiquark scatterings at the tree level are shown in figure 2 [3]. A lot of processes are related to annihilation and creation of quark and antiquark. An exchange of the two initial (final) quarks needs to be considered if the quarks have the same flavor. Quark-antiquark-antiquark elastic scatterings are similar to quark-quark-antiquark elastic scatterings.

Because of triple-gluon couplings and four-gluon couplings, the 3-to-3 gluon elastic scatterings are complicated. Four typical processes at the lowest order are exhibited in figure 3 [4]. Only the diagram $B_{\sim\sim}$ involves two four-gluon couplings. The other three diagrams include one four-gluon coupling and two triple-gluon couplings. The diagram B_U has the four-gluon coupling related to two final gluons, $B_{\sim+}$ to two final gluons and one initial gluon, and $B_{\sim-4(56)}$ to one exchanged final gluon and two initial gluons.

Except for the diagram $B_{\sim\sim}$ of which the squared amplitude is calculable by hand, we make fortran codes in Feynman gauge to derive the squared amplitudes of other diagrams that indicate the elastic scatterings of $qqq \rightarrow qqq$, $qq\bar{q} \rightarrow qq\bar{q}$, $q\bar{q}\bar{q} \rightarrow q\bar{q}\bar{q}$ and $ggg \rightarrow ggg$, respectively. While the squared amplitudes for the 3-to-3 elastic scatterings are used in the transport equation, the quark and gluon propagators are regularized with a screening mass [5] to remove Coulomb exchange divergence.

3. Numerical solutions and summary

Anisotropic parton momentum distributions are formed in initial central Au-Au collisions. Such anisotropy can be eliminated by elastic scatterings among partons. For a central Au-Au collision at $\sqrt{s_{NN}} = 200$ GeV, we obtain from HIJING Monte Carlo simulation [6, 7] the initial gluon distribution that corresponds to a gluon number density of 19.4 fm^{-3} ,

$$f(\vec{p}) = \frac{17.1(2\pi)^{1.5}}{g_G \pi R_A^2 Y(|\vec{p}| / \cosh(y) + 0.3)} e^{-|\vec{p}|/(0.9 \cosh(y)) - (|\vec{p}| \tanh(y))^2/8} \quad (3)$$

where $R_A = 6.4 \text{ fm}$, $g_G = 16$ and the rapidity $-5 \leq y \leq 5$. Any of the up-quark, down-quark, up-antiquark and down-antiquark initial distribution functions is assumed to be one sixth of the gluon initial distribution function. Starting from the time t_{ini} , when anisotropic quark-gluon matter is formed and ending at the time t_{iso} , when local momentum isotropy is established, the transport equation is solved. Local momentum isotropy, in other words, a thermal state is obtained if distribution function curves in different directions finally overlap through the thermalization time $t_{\text{iso}} - t_{\text{ini}}$. For quark matter that is governed by the $qq \rightarrow qq$ and $qqq \rightarrow qqq$ elastic scatterings, thermalization time is about $1.8 \text{ fm}/c$; for quark matter interacted by antiquark matter via $q\bar{q} \rightarrow q\bar{q}$, $qq\bar{q} \rightarrow qq\bar{q}$ and $q\bar{q}\bar{q} \rightarrow q\bar{q}\bar{q}$ elastic scatterings, thermalization time is about $1.55 \text{ fm}/c$; for gluon matter the $gg \rightarrow gg$ and $ggg \rightarrow ggg$ elastic scatterings give a thermalization time of the order of $0.45 \text{ fm}/c$. While the $qq \rightarrow qq$ and $qqq \rightarrow qqq$ elastic scatterings give

a long thermalization time, the $q\bar{q} \rightarrow q\bar{q}$, $qq\bar{q} \rightarrow qq\bar{q}$ and $q\bar{q}\bar{q} \rightarrow q\bar{q}\bar{q}$ elastic scatterings apparently shorten thermalization time of quark matter.

In summary, the larger the squared amplitudes for the 2-to-2 and 3-to-3 elastic scatterings are, the faster thermalization goes. $|\mathcal{M}_{3\rightarrow 3}|^2$ for quark matter gets larger and larger if we do a successive inclusion of quark-quark-quark, quark-quark-antiquark and quark-antiquark-antiquark elastic scatterings; correspondingly, thermalization time of quark matter becomes shorter and shorter. Rapid thermalization of gluon matter is obtained from the 2-to-2 and 3-to-3 gluon elastic scatterings. The study of thermalization leads to the importance of multi-parton elastic scatterings [8] in addition to the 2-body parton scatterings [9, 10, 11, 12]. We note here that we have not studied the contributions of $qg \rightarrow qg$, $qqg \rightarrow qqg$, $q\bar{q}g \rightarrow q\bar{q}g$ and $qgg \rightarrow qgg$ since our strategy is that we first give an independent study of gluon matter thermalization or of quark matter thermalization and then study the influence of the interplay of quark matter, antiquark matter and gluon matter. Because the derivation of the squared amplitude for a 3-to-3 elastic scattering is complicated and very time-consuming and numerically solving the transport equation needs months, we have to study the 3-to-3 elastic scatterings one by one. But the contributions of $qg \rightarrow qg$, $qqg \rightarrow qqg$, $q\bar{q}g \rightarrow q\bar{q}g$ and $qgg \rightarrow qgg$ are expected to shorten thermalization time of quark matter since they increase both $|\mathcal{M}_{2\rightarrow 2}|^2$ and $|\mathcal{M}_{3\rightarrow 3}|^2$, and the contributions will be considered in the coming years.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 10675079.

References

- [1] Reichl L E 1980 *A Modern Course in Statistical Physics* (Austin: University of Texas Press)
- [2] Xu X.-M., Peng R and Weber H J 2005 *Phys. Lett.* **B629** 68
- [3] Xu X.-M., Ma C.-C., Chen A.-Q. and Weber H J 2007 *Phys. Lett.* **B645** 146
- [4] Xu X.-M., Sun Y, Chen A.-Q. and Zheng L 2004 *Nucl. Phys.* **A744** 347
- [5] Biró T S, Müller B and Wang X.-N. 1992 *Phys. Lett.* **B283** 171
- [6] Wang X.-N. and Gyulassy M 1991 *Phys. Rev.* **D44** 3501
Gyulassy M and Wang X.-N. 1994 *Comput. Phys. Commun.* **83** 307
Wang X.-N. 1997 *Phys. Rep.* **280** 287
- [7] Lévai P, Müller B and Wang X.-N. 1995 *Phys. Rev.* **C51** 3326
- [8] Liu W and Ko C M 2006 Effect of three-body elastic scattering on heavy quark momentum degradation in the quark-gluon plasma *Preprint* nucl-th/0603004.
- [9] Shin G R and Müller B 2003 *J. Phys.* **G29** 2485
- [10] Xu Z and Greiner C 2005 *Phys. Rev.* **C71** 064901
- [11] Wong S M H 1996 *Phys. Rev.* **C54** 2588
- [12] Nayak G C, Dumitru A, McLerran L and Greiner W 2001 *Nucl. Phys.* **A687** 457

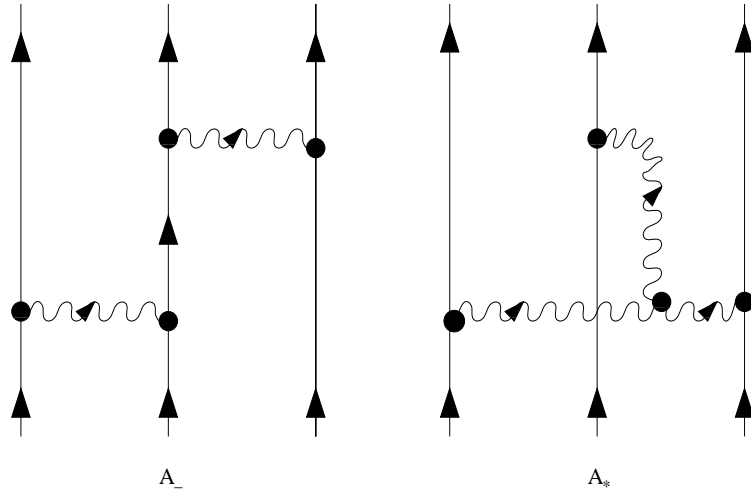


Figure 1. Quark-quark-quark elastic scatterings.

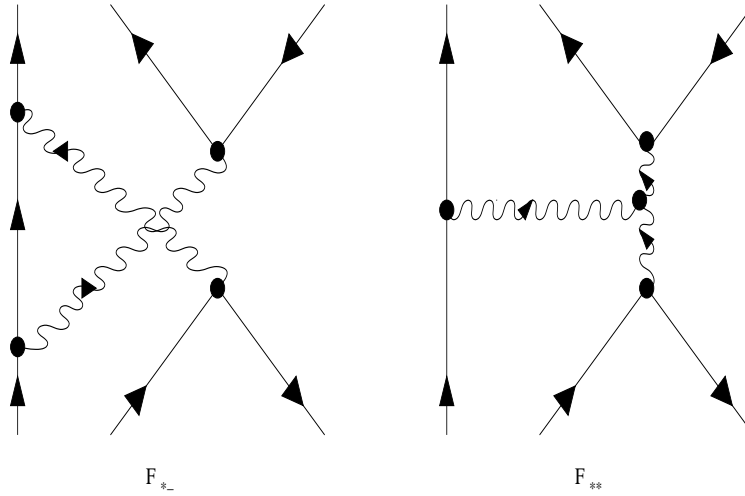


Figure 2. Quark-quark-antiquark elastic scatterings.

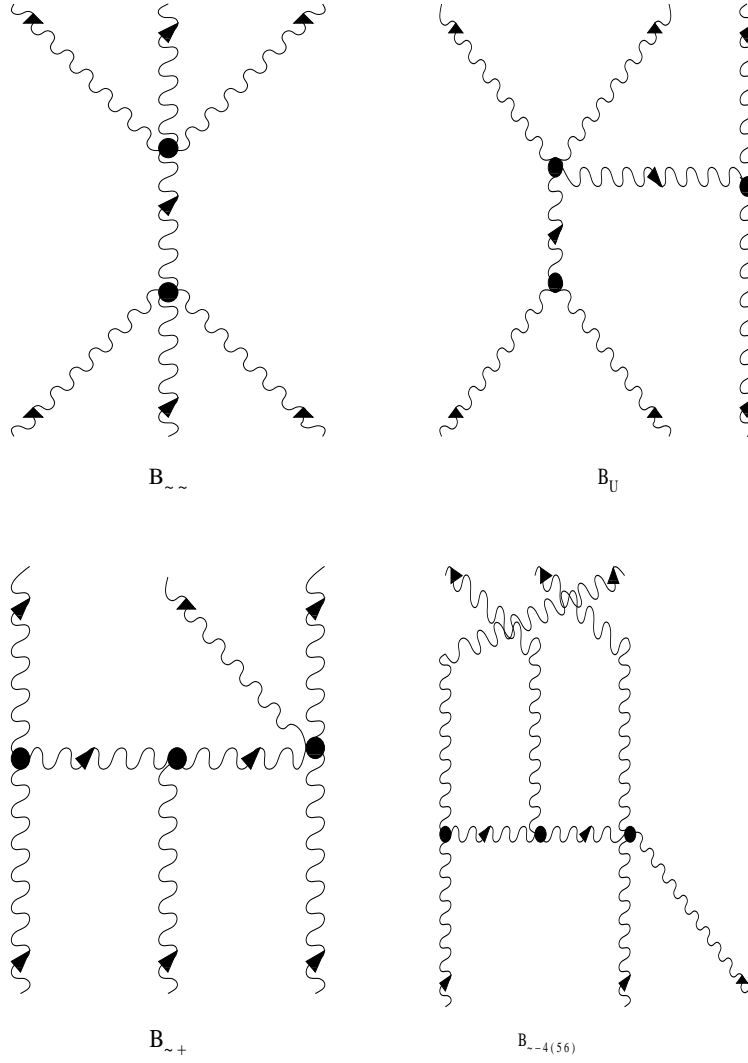


Figure 3. Gluon-gluon-gluon elastic scatterings.